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CP violation in the 3 jet and 4 jet decays of the Z boson at GigaZ

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Abstract. We review CP-violating effects in $Z \to 3$ jet and $Z \to 4$ jet decays, assuming the presence of CP-violating effective $Zb\bar{b}G$ and $Zb\bar{b}GG$ couplings. Longitudinal beam polarization is included in the studies. We propose a direct search for such CP-violating couplings by using various CP-odd observables. The data of a future linear collider running at the Z-resonance in the so-called GigaZ option should give significant information on the couplings. Finally we show that stringent bounds on the mass of excited b quarks can be derived if appropriate couplings are of a size characteristic of a strong interaction.

1 Introduction

One of the most promising projects in today's high energy physics is an electron–positron linear collider, for example TESLA [1]. At such a linear collider one should be able to polarize the electrons with the same technology as at the SLC to up to 80%. At TESLA it should also be possible to run with positrons polarized up to 60% [1]. With a luminosity of $\mathcal{L} \simeq 5 \cdot 10^{33}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ at energies close to the peak of the Z-resonance TESLA could produce 10^9 Z bosons in about 70 days of running [1]. In this scenario, often referred to as GigaZ, the measurements already performed at the electron–positron collider experiments at LEP and SLC could be redone with increased precision.

An interesting topic is the test of the CP-symmetry in Z decays. There is already a number of theoretical ([2–21] and references therein) and experimental [22–30] studies of this subject. In the present paper we will study a flavor-diagonal Z decay where CP-violating effects within the standard model (SM) are estimated to be very small [4]. Thus, looking for CP violation in such Z decays means looking for new physics beyond the SM.

For a model-independent systematic analysis of CP violation in Z decays we use the effective Lagrangian approach as described in [4, 9]. Of particular interest are Z decays involving heavy leptons or quarks. Thus, the process $Z \to b\bar{b}G$, which is sensitive to effective CP-violating couplings in the $Zb\bar{b}G$ vertex, has been analyzed theoretically in [15, 17] and experimentally in [24]. No significant deviation from the SM has been found.

If CP-violating couplings are introduced in the $Zb\bar{b}G$ vertex, they will, because of gauge invariance of QCD, appear in the $Zb\bar{b}GG$ vertex as well. But the $Zb\bar{b}GG$ vertex could in principle contain new coupling parameters. The analysis of the 4 jet decays of the Z boson involving b quarks looks into both, 4- and 5-point vertices. This has been investigated theoretically in [20] and experimentally in [30]. Also in this case no significant deviation from the SM has been found.

In this paper we review the results of the calculations of the processes $Z \to 3$ jets and $Z \to 4$ jets including CP-violating couplings, with at least two of the jets originating from a b or $\bar{b}$ quark, for the GigaZ scenario assuming longitudinal beam polarization for electrons and positrons. All details of the calculation for unpolarized e^+, e^- beams can be found in [15, 17, 20, 31].

Finally we make an estimate for models with excited quarks and show that one can obtain stringent bounds on their mass. This, however, requires the introduction of a new type of strong interactions for quarks.

In Sect. 2 we briefly review the theoretical framework of our computations. Models with excited quarks are discussed in Sect. 3. Next, in Sect. 4, we define CP-odd tensor and vector observables and calculate their sensitivities to anomalous couplings. Achievable bounds on the mass of excited quarks are presented. Our conclusions can be found in Sect. 5.

2 Effective Lagrangian approach

For a model-independent study of CP violation in 3 jet and 4 jet decays of the Z boson we use the effective Lagrangian approach as explained in [4]. We could add to the SM Lagrangian $\mathcal{L}_{\mathrm{SM}}$ a CP-violating term $\mathcal{L}_{CP}$ containing all CP-odd local operators with a mass dimension $d \leq 6$ (*after* electroweak symmetry breaking) that can be constructed with SM fields. However, it turns out that quite a number of such coupling terms can contribute to the reac-

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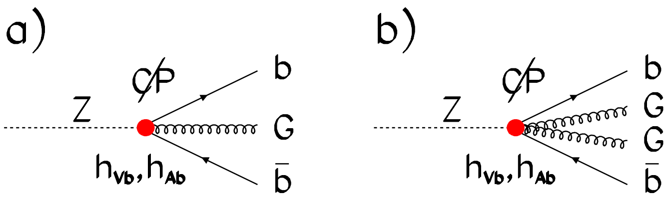

Fig. 1a,b. The CP-violating vertices

tions analyzed here. To keep the analysis manageable we restrict ourselves to coupling terms involving the Z and the b quarks and in addition any number of gluons. Then the effective CP-violating Lagrangian with $d \leq 6$ relevant to our analysis is

$$\mathcal{L}_{CP}(x) = -\frac{\mathrm{i}}{2}\widetilde{d}_b \bar{b}(x)\sigma^{\mu\nu}\gamma_5 b(x)\left[\partial_\mu Z_\nu(x) - \partial_\nu Z_\mu(x)\right] + \left[h_{Vb}\bar{b}(x)T^a\gamma^\nu b(x) + h_{Ab}\bar{b}(x)T^a\gamma^\nu\gamma_5 b(x)\right] \times Z^\mu(x) G^a_{\mu\nu}(x)\,, \tag{1}$$

where $b(x)$ denotes the b quark field, $Z^\mu(x)$ and $G^a_{\mu\nu}(x)$ represent the field of the Z boson and the field strength tensor of the gluon, respectively, and $T^a = \lambda^a/2$ are the generators of $SU(3)_C$ [32]. In (1) $\widetilde{d}_b$ is the weak dipole moment and h_{Vb}, h_{Ab} are CP-violating vector and axial vector chirality conserving coupling constants. As effective coupling constants in $\mathcal{L}_{CP}$ the parameters $\widetilde{d}_b$, h_{Vb}, h_{Ab} are real. They are related to form factors of vertices but should not be confused with the latter (cf. e.g. [18]).

Information on the spin of the final state partons is hardly available experimentally. Thus, we consider as observables only the parton's energies and momenta. Then, effects linear in the dipole form factor $\widetilde{d}_b$ are suppressed by powers of m_b/m_Z. So angular correlations of the jets in $Z \to 3$ jets and $Z \to 4$ jets are only sensitive to the couplings h_{Vb} and h_{Ab}.

The corresponding vertices following from $\mathcal{L}_{CP}$ are shown in Fig. 1. Because the non-abelian field strength tensor has a term quadratic in the gluon fields the $Zb\bar{b}G$- and $Zb\bar{b}GG$-vertices are related.

We define dimensionless coupling constants $\widehat{h}_{Vb,Ab}$ using the Z mass as the scale parameter by

$$h_{Vb,Ab} = \frac{e g_s}{\sin\vartheta_\mathrm{W}\cos\vartheta_\mathrm{W} m_Z^2}\widehat{h}_{Vb,Ab}\,, \tag{2}$$

where $e = \sqrt{4\pi\,\alpha}$, $g_s = \sqrt{4\pi\,\alpha_\mathrm{s}}$ and ϑ_W is the weak mixing angle. For numerical calculations we set $m_Z = 91.187\,\mathrm{GeV}$, $\sin^2\vartheta_\mathrm{W} = 0.2236$ and the fine structure constant and α_s at the Z mass to $\alpha = 1/128.9$ and $\alpha_\mathrm{s} = 0.118$ [33]. Our calculations are carried out in leading order of the CP-violating couplings of $\mathcal{L}_{CP}$ and the SM couplings. A non-vanishing b quark mass of 4.5 GeV is included[1]; the masses of the u, d, s, c quarks are neglected.

[1] We use here the pole mass value for the b quark. In our leading order calculation we could as well use the running b mass at m_Z: $m_b(m_Z) \simeq 3\,\mathrm{GeV}$ [34]. This would result only in minimal changes in our correlations.

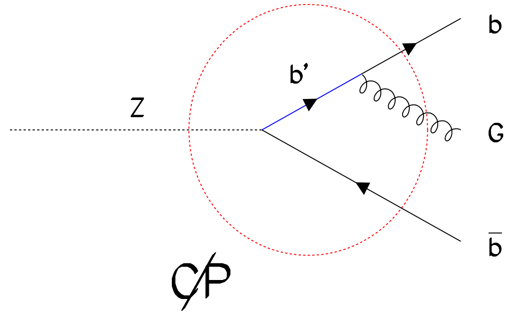

Fig. 2. Contribution to $Zb\bar{b}G$ from an excited quark b'. The diagram with the role of b and $\bar{b}$ exchanged is not shown

3 Models with excited quarks

In this section we discuss the possible generation of chirality conserving CP-violating interactions as introduced in the previous section in models with excited quarks. Excitations of quarks would be natural in a scenario where quarks have substructure and participate in a new type of strong interaction. This type of models and effects from excited quarks at hadron colliders have for instance been discussed in [35]. In particular, we assume here that b quarks have excited partners b', which could have spin $\frac{1}{2}$ or $\frac{3}{2}$. For simplicity we consider a b' of spin $\frac{1}{2}$ and mass $m_{b'}$. Due to color gauge invariance we expect the $bb'G$ couplings to be chirality-flipping dipole couplings. Then, couplings $\widehat{h}_{Vb,Ab}$ as introduced in (1) can be generated by the following effective interactions of b' and b quarks, Z bosons and gluons:

$$\mathcal{L}'(x) = -\frac{e}{2\sin\vartheta_\mathrm{W}\cos\vartheta_\mathrm{W}} Z_\mu(x)\bar{b}'(x)\gamma^\mu(g'_V - g'_A\gamma_5)b(x) - \mathrm{i}\frac{g_s}{2m_{b'}}\hat{d}_c \bar{b}'(x)\sigma^{\mu\nu}\gamma_5 T^a b(x) G^a_{\mu\nu}(x) + \mathrm{h.c.} \tag{3}$$

Here g'_V, g'_A and $\hat{d}_c$ are complex parameters, which can be expected to be of order one if the underlying dynamics is strongly interacting. In addition to $\hat{d}_c$, the chromoelectric dipole transition form factor $b \to b'$, there will be in general also a chromomagnetic transition form factor $\hat{d}_m$ which is omitted here for brevity.

The couplings $\widehat{h}_{Vb,Ab}$ have been calculated [16] in this model from the diagrams of the type shown in Fig. 2 for $m_{b'} \gg m_Z$:

$$\widehat{h}_{Vb} = \frac{m_Z^2}{m_{b'}^2}\mathrm{Re}(\hat{d}_c g'^{*}_A)\,, \qquad \widehat{h}_{Ab} = -\frac{m_Z^2}{m_{b'}^2}\mathrm{Re}(\hat{d}_c g'^{*}_V)\,. \tag{4}$$

4 Study of CP-violating couplings

In our study we assume that one is able to flavor-tag the b quarks and to measure their momenta. This is justified due to the extremely good b-tagging capabilities foreseen at TESLA [1]. For instance, the impact parameter resolution at TESLA is expected to be about a factor 10 better than at LEP [36].

The definition of a 3 and 4 jet sample requires the introduction of resolution cuts. We use JADE cuts [37] requiring

$$y_{ij} = \frac{2\,E_i E_j\,(1-\cos\vartheta_{ij})}{m_Z^2} > y_{\rm cut}\ , \tag{5}$$

with ϑ_{ij} the angle between the momentum directions of any two partons ($i \neq j$) and E_i, E_j their energies in the Z rest system.

4.1 CP-odd tensor and vector observables

We study our CP-violating couplings using CP-odd observables constructed from the momentum directions of the b and $\bar{b}$ quarks, $\widehat{\mathbf{k}}_b = \mathbf{k}_b/|\mathbf{k}_b|$ and $\widehat{\mathbf{k}}_{\bar{b}} = \mathbf{k}_{\bar{b}}/|\mathbf{k}_{\bar{b}}|$ (cf. [4,9,11,17]):

$$T_{ij} = (\widehat{\mathbf{k}}_{\bar{b}} - \widehat{\mathbf{k}}_b)_i(\widehat{\mathbf{k}}_{\bar{b}} \times \widehat{\mathbf{k}}_b)_j + (i \leftrightarrow j)\ , \tag{6}$$

$$V_i = (\widehat{\mathbf{k}}_{\bar{b}} \times \widehat{\mathbf{k}}_b)_i\ , \tag{7}$$

with i, j the Cartesian vector indices in the Z rest system.

The observables T_{ij} transform as tensor components, V_i as vector components. For polarized e^+e^- beams and our rotationally invariant cuts (5) their expectation values are then proportional to the Z tensor polarization S_{ij} and vector polarization s_i, respectively. For all definitions concerning the Z density matrix see Sect. 2.1 of [4]. Defining the positive z-axis in the e^+ beam direction, we have

$$\mathbf{s} = \begin{pmatrix} 0 \\ 0 \\ s_3 \end{pmatrix}\ , \tag{8}$$

$$(S_{ij}) = \frac{1}{6}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}\ , \tag{9}$$

where

$$s_3 = \frac{s_3^{(0)}(1-P_+P_-) + (P_+ - P_-)}{(1-P_+P_-) + s_3^{(0)}(P_+ - P_-)} \tag{10}$$

and

$$s_3^{(0)} = \frac{2\,g_{Ve}g_{Ae}}{g_{Ve}^2 + g_{Ae}^2} = 0.209\ , \tag{11}$$

with $g_{Ve} = -1/2 + 2\sin^2\vartheta_{\rm W}$ and $g_{Ae} = -1/2$ the weak vector and axial vector Zee couplings. P_+ and P_- are the longitudinal polarizations for positron and electron, respectively, measured in the direction of the particle's velocity. We have $|P_\pm| \leq 1$. From (8)–(11) we see that the components T_{33} and V_3 are the most sensitive ones.

Note that the tensor observables do *not* change their sign upon charge misidentification ($\widehat{\mathbf{k}}_{\bar{b}} \leftrightarrow \widehat{\mathbf{k}}_b$) whereas the vector observables do. Thus, it is only for the measurement of the latter that charge identification is indispensable.

We have computed the expectation values of the observables (6) and (7) for different JADE cuts (5), as a function of

$$\widehat{h}_b = \widehat{h}_{Ab}g_{Vb} - \widehat{h}_{Vb}g_{Ab} \tag{12}$$

and

$$\widetilde{h}_b = \widehat{h}_{Vb}g_{Vb} - \widehat{h}_{Ab}g_{Ab}\ , \tag{13}$$

where

$$g_{Vb} = -\frac{1}{2} + \frac{2}{3}\sin^2\vartheta_{\rm W}\ , \quad g_{Ab} = -\frac{1}{2}\ . \tag{14}$$

The expectation value of a CP-odd observable $\mathcal{O}$ has the following general form:

$$\langle\mathcal{O}\rangle = \left(c_1\widehat{h}_b + c_2\widetilde{h}_b\right)\frac{\Gamma^{\rm SM}_{3/4{\rm jets}}}{\Gamma_{3/4{\rm jets}}}\ , \tag{15}$$

where $\Gamma^{\rm SM}_{3/4{\rm jets}}$ and $\Gamma_{3/4{\rm jets}}$ denote the corresponding $Z \to$ 3 jets and $Z \to$ 4 jets decay widths in the SM and in the theory with SM plus CP-violating couplings, respectively. In an experimental analysis $\Gamma^{\rm SM}_{3/4{\rm jets}}$ should be taken from the theoretical calculation, $\Gamma_{3/4{\rm jets}}$ and $\langle\mathcal{O}\rangle$ from the experimental measurement. The quantity $\langle\mathcal{O}\rangle \cdot \Gamma_{3/4{\rm jets}}$ is then an observable strictly linear in the anomalous couplings.

From the measurement of a single observable (15) we can get a simple estimate of its sensitivity to $\widehat{h}_b$ by assuming $\widetilde{h}_b = 0$. The error on a measurement of $\widehat{h}_b$ is then to leading order in the anomalous couplings:

$$\delta\widehat{h}_b = \frac{\sqrt{\langle\mathcal{O}^2\rangle_{\rm SM}}}{|c_1|\sqrt{N}}\ , \tag{16}$$

where N is the number of events within cuts. Similarly, assuming $\widehat{h}_b = 0$ we get the error on $\widetilde{h}_b$ as

$$\delta\widetilde{h}_b = \frac{\sqrt{\langle\mathcal{O}^2\rangle_{\rm SM}}}{|c_2|\sqrt{N}}\ . \tag{17}$$

A measure for the sensitivity of $\mathcal{O}$ to $\widehat{h}_b$ ($\widetilde{h}_b$) is then $1/\delta\widehat{h}_b$ ($1/\delta\widetilde{h}_b$).

In very good approximation, it was found for $Z \to$ 3 jets and $Z \to$ 4 jets that the tensor observables are only sensitive to $\widehat{h}_b$ and the vector observables only to $\widetilde{h}_b$. A detailed discussion of this can be found in [15,17,20,31].

A measurement of $\widehat{h}_b$, $\widetilde{h}_b$ has to produce a mean value larger than $\delta\widehat{h}_b$ (16), $\delta\widetilde{h}_b$ (17) to be able to claim a non-zero effect at the 1 s.d. level.

4.2 Numerical results

We have calculated the sensitivities to $\widehat{h}_b$ and $\widetilde{h}_b$ for the tensor (6) and vector (7) observables varying the jet resolution parameter $y_{\rm cut}$. Comparing with optimal observables it was found for unpolarized beams [17,20] that these

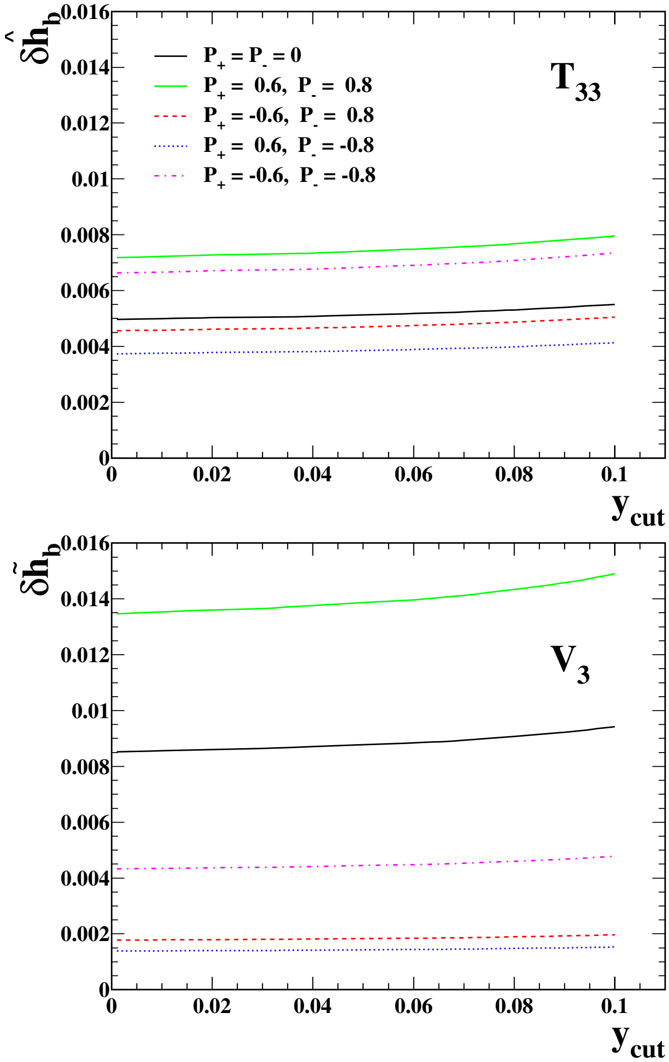

Fig. 3. The inverse sensitivities of tensor T_{33} and vector V_3 observables to $\widehat{h}_b$ and $\widetilde{h}_b$ (12) and (13) obtainable in $Z \to 3$ jets, as a function of the jet resolution parameter y_{cut} (5) for different longitudinal polarizations of the e^+ and e^- beams assuming an integrated luminosity which would lead to 10^9 Z decays without polarization

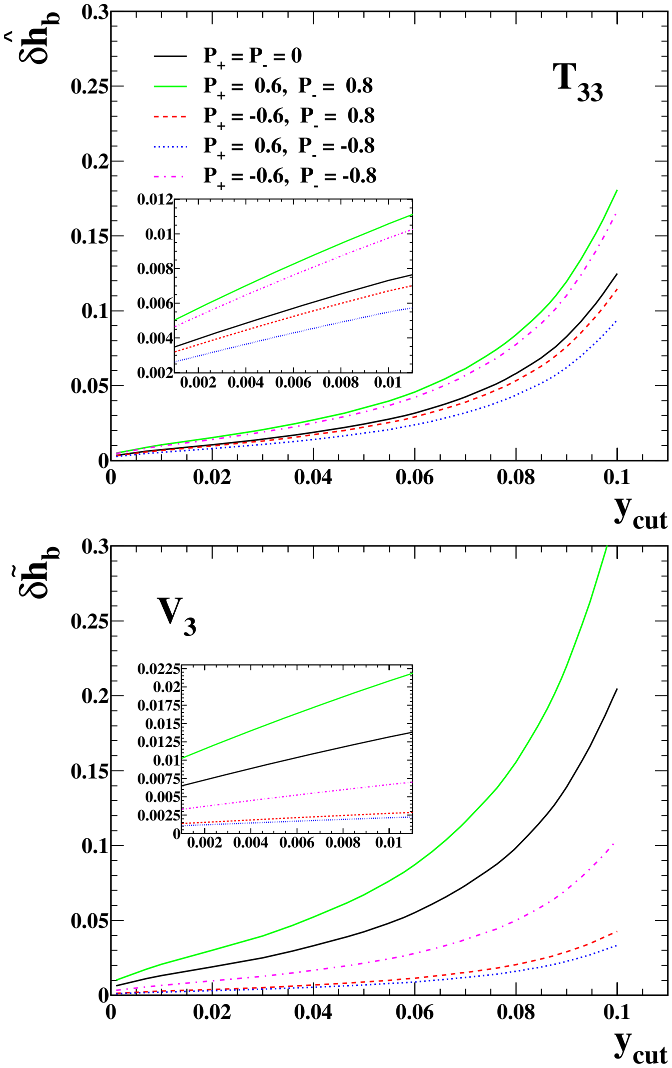

Fig. 4. The inverse sensitivities of tensor T_{33} and vector V_3 observables to $\widehat{h}_b$ and $\widetilde{h}_b$ (12) and (13) obtainable in $Z \to 4$ jets, as a function of the jet resolution parameter y_{cut} (5) for different longitudinal polarizations of the e^+ and e^- beams assuming an integrated luminosity which would lead to 10^9 Z decays without polarization

simple observables (6) and (7) reach nearly optimal sensitivities. Therefore optimal observables are not considered in the following.

We assume a total number of $N_{\mathrm{tot}} = 10^9$ Z decays for unpolarized beams, following the GigaZ scenario. The number N of events within cuts which is available for the analysis is then given by

$$N_{3/4\mathrm{jets}} = N_{\mathrm{tot}} \frac{\Gamma^{\mathrm{SM}}_{3/4\mathrm{jets}}}{\Gamma_Z} , \tag{18}$$

with Γ_Z being the total Z decay width. Solely due to higher statistics in GigaZ of about a factor of one hundred compared to the sum of the four LEP experiments, the sensitivity to the CP-violating couplings increases by a factor 10, as can be seen from (16) and (17).

The inverse sensitivities to these CP-odd couplings as calculated from (16) and (17), respectively, are shown in Fig. 3 for $Z \to 3$ jets and in Fig. 4 for $Z \to 4$ jets for different longitudinal beam polarizations. The sensitivity decreases with increasing y_{cut} for all observables due to the decrease in number of events available.

Because the expectation value of the tensor observable does not depend on the longitudinal polarization (9), the differences in $\delta\widehat{h}_b$ for different polarization choices reflect only the change in statistics. For $P_+ = 0.6$ and $P_- = -0.8$ the enhancement of the Z production rate is largest. The differences in $\delta\widetilde{h}_b$ reflect both the change in statistics and the modification of the expectation value due to the polarization (10). For $P_+ = 0.6$ and $P_- = -0.8$ the sensitivity increases by more than a factor of six compared to unpolarized beams. A convenient choice of the polarizations can

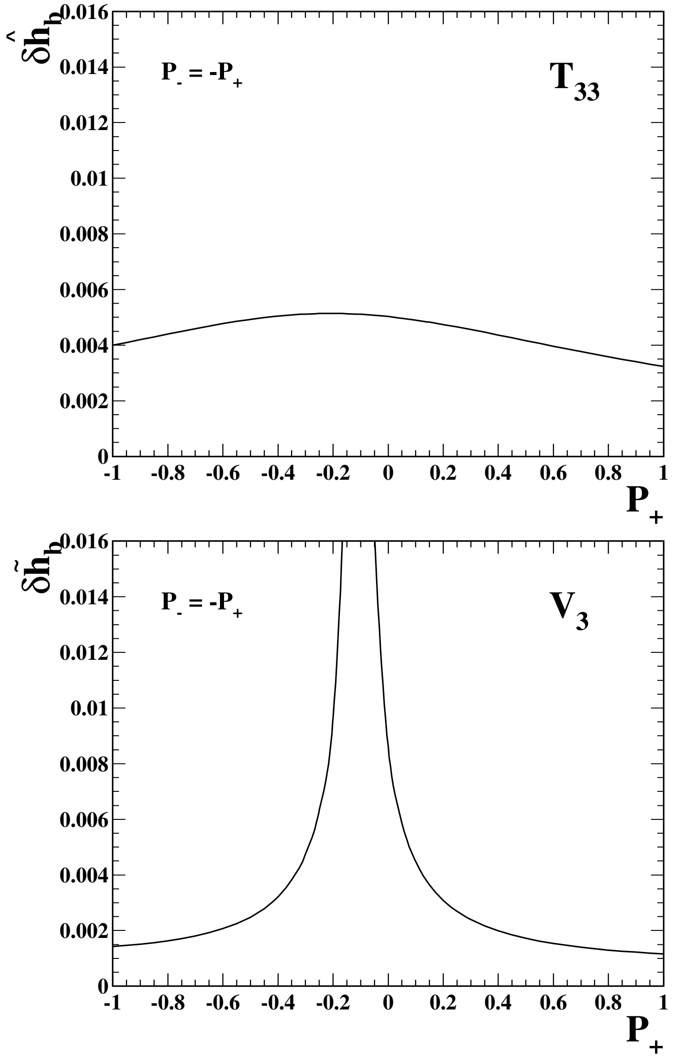

Fig. 5. The inverse sensitivities of tensor T_{33} and vector V_3 observables to $\widehat{h}_b$ and $\widetilde{h}_b$ (12) and (13) obtainable in $Z \to 3$ jets for $y_{\text{cut}} = 0.02$, as a function of the e^+ beam polarization for $P_- = -P_+$ assuming an integrated luminosity which would lead to 10^9 Z decays without polarization

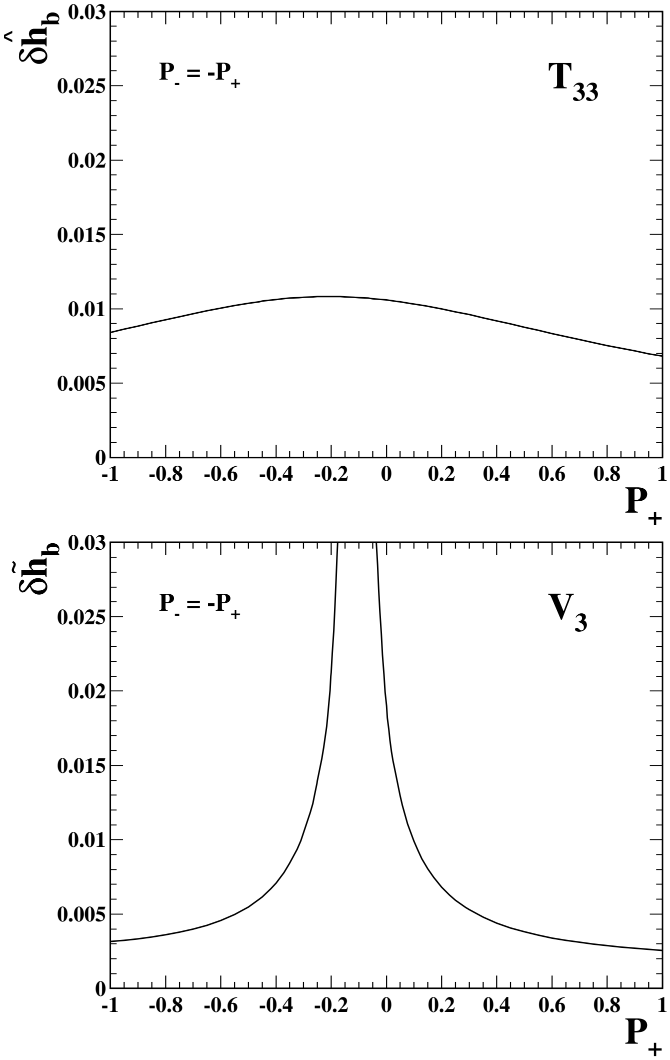

Fig. 6. The inverse sensitivities of tensor T_{33} and vector V_3 observables to $\widehat{h}_b$ and $\widetilde{h}_b$ (12) and (13) obtainable in $Z \to 4$ jets for $y_{\text{cut}} = 0.02$, as a function of the e^+ beam polarization for $P_- = -P_+$ assuming an integrated luminosity which would lead to 10^9 Z decays without polarization

even lead to a better sensitivity of the vector observable to $\widetilde{h}_b$ than of the tensor observable to $\widehat{h}_b$.

In contrast, an unsuitable choice of the polarizations could kill any sensitivity of the vector observable. This is illustrated in Figs. 5 and 6 for $Z \to 3$ jets and $Z \to 4$ jets, respectively: The inverse sensitivities are shown as a function of the positron polarization assuming $P_- = -P_+$. For $P_+ \simeq -0.1$ the expectation value for the vector observable and therefore the sensitivity to $\widetilde{h}_b$ vanishes. For the tensor observable this cannot happen because the sensitivity to $\widehat{h}_b$ depends on the polarization only due to the change in the total number of Z decays.

4.3 Interpretation in the framework of excited quarks

If a measurement of $\widehat{h}_b$, $\widetilde{h}_b$ produces a mean value lower than $\delta\widehat{h}_b$ (16), $\delta\widetilde{h}_b$ (17) a non-zero effect at the 1 s.d. level cannot be claimed and therefore an upper limit on these couplings can be derived. Using (4) this can be translated into lower bounds on the excited quark mass $m_{b'}$. Assuming $\text{Re}(\hat{d}_c g_A'^*) = \text{Re}(\hat{d}_c g_V'^*) = 1$ these bounds are shown in Fig. 7 for $Z \to 3$ jets and in Fig. 8 for $Z \to 4$ jets for different longitudinal beam polarizations.

In [38] at the 95% confidence level excited quarks with mass between 80 and 570 GeV and between 580 and 760 GeV were excluded. In [39] the lower limit $m_{q'} > 775$ GeV on the masses of the excited quarks was given. However, these results apply to excited u and d quarks only and do not exclude a lighter b' quark.

5 Conclusions

In this paper, we have reviewed calculations concerning the search for CP violation in the 3 jet and 4 jet decays of

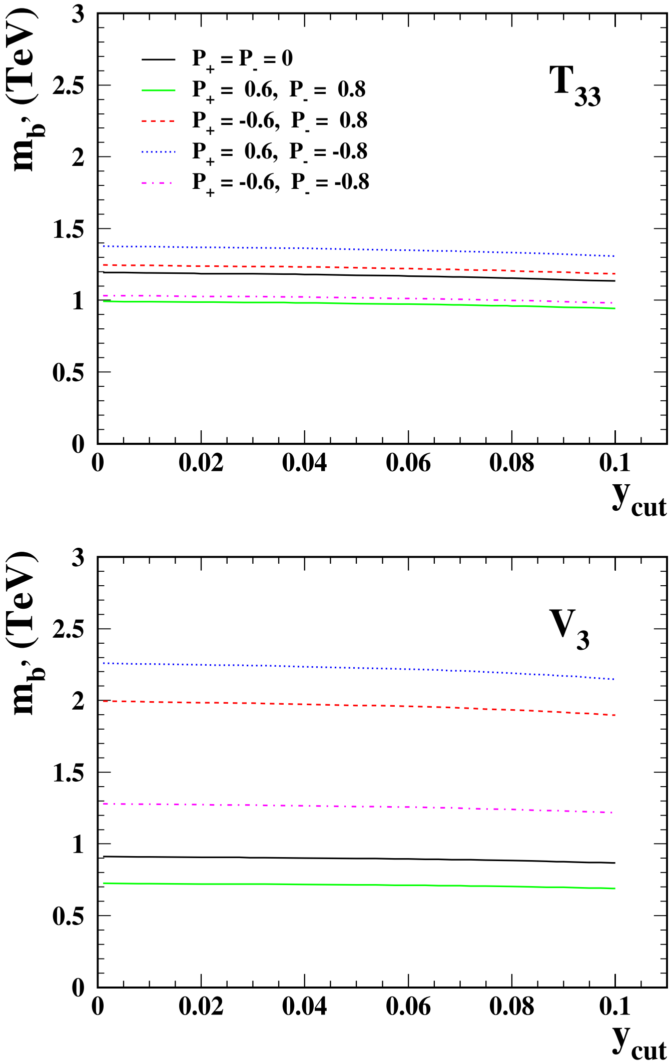

Fig. 7. Lower limits on the excited quark mass $m_{b'}$ at the 1 s.d. level which can be derived from a measurement of tensor T_{33} and vector V_3 observables in $Z \to 3$ jets, as a function of the jet resolution parameter $y_{\rm cut}$ (5) for different longitudinal polarizations of the e^+ and e^- beams assuming an integrated luminosity which would lead to 10^9 Z decays without polarization. Couplings for the b' as discussed in the text are assumed

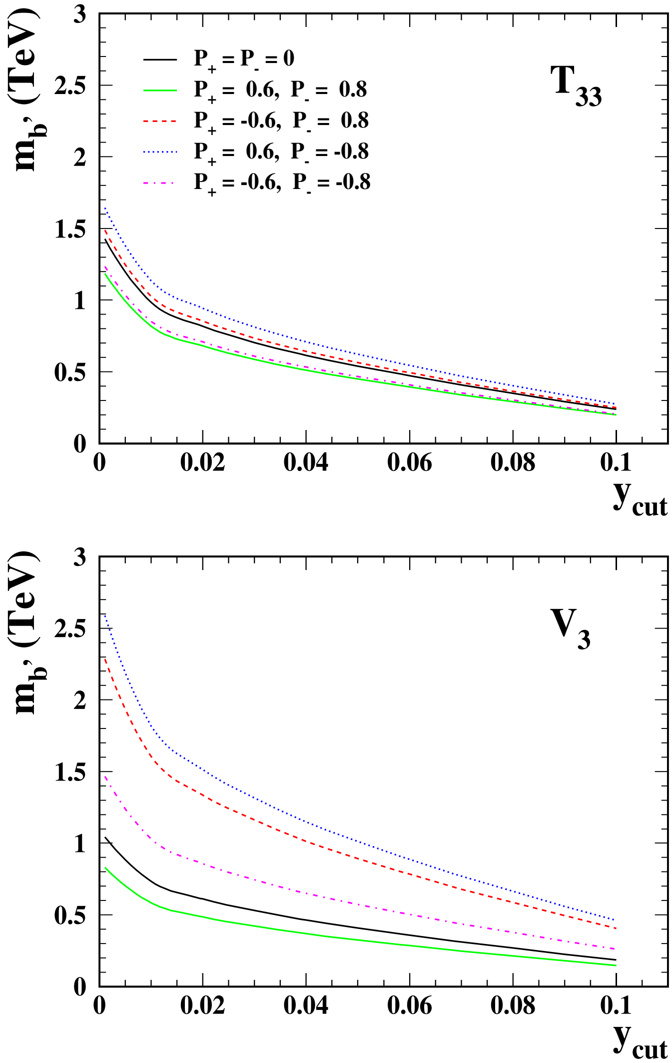

Fig. 8. Lower limits on the excited quark mass $m_{b'}$ at the 1 s.d. level which can be derived from a measurement of tensor T_{33} and vector V_3 observables in $Z \to 4$ jets, as a function of the jet resolution parameter $y_{\rm cut}$ (5) for different longitudinal polarizations of the e^+ and e^- beams assuming an integrated luminosity which would lead to 10^9 Z decays without polarization. Couplings for the b' as discussed in the text are assumed

the Z boson with at least two of the jets originating from b and $\bar{b}$ quarks. We have studied a CP-violating contact interaction with a vector and axial vector coupling $\widehat{h}_{Vb}$, $\widehat{h}_{Ab}$ (1), (2). We have discussed how such couplings can be generated in models with an excited b quark, b'. Such couplings can also arise at one loop level in multi-Higgs extensions of the standard model [16, 40]. Longitudinal beam polarization is included.

We studied a tensor and vector observable which can be used for the measurement of the anomalous couplings. While the sensitivity of the tensor observable to CP-violating effects is only affected by the variation of statistics due to beam polarization given a certain integrated luminosity, the expectation value of the vector observable itself changes.

If flavor tagging of b and $\bar{b}$ jets is available then, with a total number of 10^9 Z decays and choosing a cut parameter $y_{\rm cut} = 0.02$, the anomalous coupling constant $\widehat{h}_b$ (12) can be determined with an accuracy of order 0.004 ($Z \to 3$ jets) and 0.008 ($Z \to 4$ jets) at the 1 s.d. level using the tensor observable T_{33} (6) for the measurement. Here, the b–$\bar{b}$ distinction is not necessary. These accuracies are close to the ones which already can be obtained for unpolarized beams. If in a measurement a non-zero effect at the 1 s.d. level is not observed, then excited quark masses $m_{b'}$ lower than 1.4 TeV ($Z \to 3$ jets) and 0.94 TeV ($Z \to 4$ jets) can be excluded if appropriate couplings are of a size characteristic of a strong interaction.

If the b–$\bar{b}$ distinction is experimentally realizable, which should be the case at a future linear collider, the coupling constant $\widetilde{h}_b$ can be measured with an accuracy of order 0.0015 ($Z \to 3$ jets) and 0.003 ($Z \to 4$ jets) using the vector observable V_3 (7) and choosing $P_+ = 0.6$ and $P_- = -0.8$ as longitudinal polarizations of positron and electron, respectively. In case of a non-observation of an effect at the 1 s.d. level excited quark masses $m_{b'}$ lower than 2.2 TeV ($Z \to 3$ jets) and 1.5 TeV ($Z \to 4$ jets) can be excluded if the appropriate couplings are of a size characteristic of a strong interaction.

Comparing 3 and 4 jet analyses we found that the sensitivity to the anomalous coupling $\widehat{h}_b$ was roughly constant as a function of the cut parameter y_{cut} for $y_{\text{cut}} < 0.1$ in the 3 jet case. For the 4 jet case the sensitivity was found to increase as y_{cut} decreases. For $y_{\text{cut}} \lesssim 0.01$ the 4 jet sensitivity was found to exceed that from 3 jets. Of course in an experimental analysis one should try to make both 3 and 4 jet analyses in order to extract the maximal possible information from the data.

In our theoretical investigations we always assumed 100% efficiencies and considered the statistical errors only. Assuming systematic errors to be of the same size as the statistical ones, the accuracies in the determinations of $\widehat{h}_b$, $\widetilde{h}_b$ discussed above should indeed be better by more than one order of magnitude than those derived from LEP. As shown in [16, 40] this will, for instance, give valuable information on the scalar sector in multi-Higgs extensions of the standard model. That interesting information on models with excited quarks can be derived as well has been discussed in detail here.

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Note added in proof: A slightly extended version of this article is available as Linear Collider note, LC-TH-2003-097 [hep-ph/0308198]. There some questions concerning electroweak gauge invariance and our Lagrangian (1) as well as next-to-leading order QCD corrections are discussed.